

Rank of Observation Matrix

$$W = M \times S$$
$$2F \times N \quad 2F \times 3 \quad 3 \times N$$

We know:

$$\text{Rank}(MS) \leq \text{Rank}(M)$$

$$\text{Rank}(MS) \leq \text{Rank}(S)$$

$$\Rightarrow \text{Rank}(MS) \leq \min(3, 2F) \quad \text{Rank}(MS) \leq \min(3, N)$$

$$\Rightarrow \text{Rank}(W) \leq \text{Rank}(MS) \leq \min(3, N, 2F)$$

Rank Theorem: $\text{Rank}(W) \leq 3$